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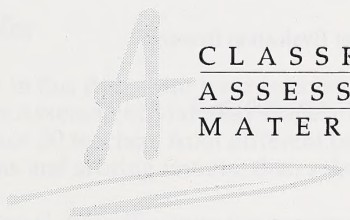
MATHEMATICS 20

EXAMPLES OF STUDENTS' RESPONSES

CLASSROOM ASSESSMENT MATERIALS

MATHEMATICS 20

EXAMPLES OF STUDENTS' RESPONSES



CLASSROOM
ASSESSMENT
MATERIALS

Alberta
EDUCATION

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Introduction

Purpose

The purpose of this document is to provide teachers, students, parents, and administrators with examples of students' responses that illustrate the provincial standards expected of students who complete Mathematics 20.

Contents

For each of the written or performance tasks, the *Examples of Students' Responses* document contains

- the student task
- the scoring criteria to be used by teachers to evaluate their students' work (these criteria can also be found in the *Teacher Manual*)
- examples of students' responses at each criteria "level"
- commentaries that illustrate and explain how the scoring criteria fit each response

Each student response in this document is reproduced as it appeared in the assessment; that is, in the students' own handwriting. Because of space limitations, the planning and drafting that may have preceded or accompanied some of the responses may not have been included. However, teachers and other readers should note that research has demonstrated a very strong relationship between the quality of planning and the degree of success in written expression.

Selection of Examples

The students' responses in this document were selected from those produced during the pilot testing of the Classroom Assessment Materials Project in May and June of 1996. A committee composed of Mathematics 20 teachers from different parts of the province reviewed and validated the assessments and scoring criteria, then selected students' responses.

These examples of students' work illustrate the provincial standards for students who complete Mathematics 20.

Considerations

Please note that

- the examples presented illustrate specific standards (scoring criteria), but are not necessarily typical of the responses submitted.
- the selected responses represent only a few of the possible approaches to each task. None of the examples is intended to serve as a model of a particular approach.
- you should consider each student example in light of the constraints of the assessment situation. Under assessment conditions, most students are able to prepare responses that must be considered as first draft only.

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In addition to this *Examples of Students' Responses*, the Mathematics 20 Classroom Assessment Materials includes a complete set of *Student Materials* and *Teacher Manual* in separate booklets.

Mathematics 20

End-of-Course Exam

- ***Written-Response Question 1***
- ***Written-Response Question 2***

CLASSROOM ASSESSMENT MATERIALS

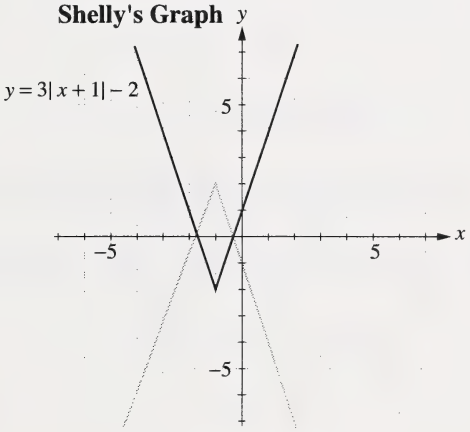
Written Response 1: Student Task

Written Response – 4 marks

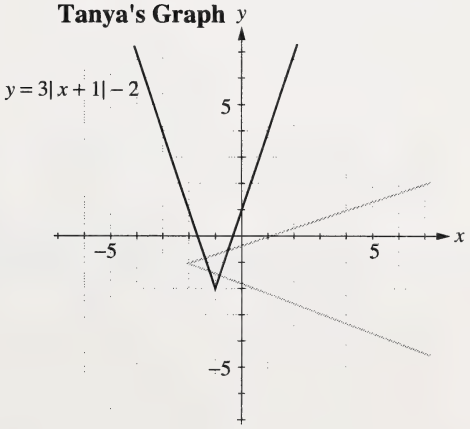
1. Shelly and Tanya are investigating the absolute value function. Both Shelly and Tanya correctly graph $y = 3|x + 1| - 2$, and then they each graph what they believe to be the inverse of this function.

Either Shelly or Tanya constructed a correct graph. Shelly concludes that the inverse of this function is a function, whereas Tanya concludes that the inverse of this function cannot be described by a single function. Identify who is correct, and explain how you reached your conclusion.

Shelly's Graph



Tanya's Graph



A Solution for Written Response 1

Tanya's graph is correct because, graphically, the inverse of a relation is the reflection of the relation across the line $y = x$. Tanya reflected $y = 3|x + 1| - 2$ across the line $y = x$, whereas Shelly reflected across the x-axis.

Scoring Criteria for Written Response 1

4 marks Tanya's graph is determined to be correct, and a correct reason is provided.

3 marks Tanya's graph is determined to be correct. An insufficient reason is provided. For example, the student describes only one point on the relation and its inverse. $((-1, -2)$ and $(-2, -1))$.

2 marks Tanya's graph is determined to be correct, and an attempt is made to provide a reason.

1 mark The student demonstrates some understanding of an inverse. For example, students make a statement like "interchange x and y."

OR

Shelly's graph is determined to be correct. The reasoning includes true statements about Shelly's graph.

0 mark The student is off topic, or leaves the paper blank.

Written Response 1: Student Responses

This response would receive a score of **4**

Scoring Criteria
- Tanya's graph is determined to be correct, and a correct reason is provided.

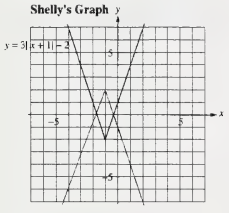
Student Task

Written Response – 4 marks

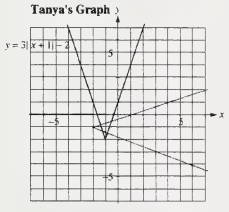
1. Shelly and Tanya are investigating the absolute value function. Both Shelly and Tanya correctly graph $y = 3|x + 1| - 2$, and then they each graph what they believe to be the inverse of this function.

Either Shelly or Tanya constructed a correct graph. Shelly concludes that the inverse of this function is a function, whereas Tanya concludes that the inverse of this function cannot be described by a single function. Identify who is correct, and explain how you reached your conclusion.

Shelly's Graph



Tanya's Graph



To find inverse of $y = 3|x+1|-2$:

Table of values for $y = 3|x+1|-2$

x	y	
−3	4	Inverse - switch x and y
−2	1	(4, −3) (1, −2) (−2, −1)
−1	−2	(1, 0) (4, 1)
0	1	
1	4	

Plot graph.

Graph equals Tanya's Graph. Tanya is correct in saying that the inverse of this function cannot be described by a single function. The conclusion was reached by the above work → finding the inverse using a table of values.

Commentary

The student
- determines Tanya's graph to be correct
- shows an acceptable reason for Tanya's graph being correct by using a table of values where the x and y coordinates are interchanged

This response would receive a score of **3**

Scoring Criteria

- Tanya's graph is determined to be correct. An insufficient reason is provided. For example, the student describes only one point on the relation and its inverse. ((–1, –2) and (–2, –1)).

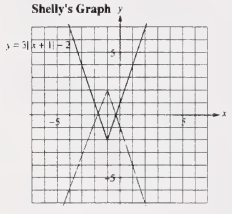
Student Task

Written Response – 4 marks

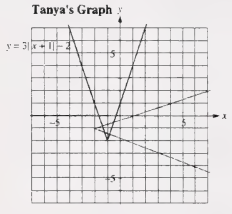
1. Shelly and Tanya are investigating the absolute value function. Both Shelly and Tanya correctly graph $y = 3|x + 1| - 2$, and then they each graph what they believe to be the inverse of this function.

   Either Shelly or Tanya constructed a correct graph. Shelly concludes that the inverse of this function is a function, whereas Tanya concludes that the inverse of this function cannot be described by a single function. Identify who is correct, and explain how you reached your conclusion.

Shelly's Graph



Tanya's Graph



I believe that tanya is correct. when you take the inverse of vertex points (-1,-2) you get (-2,-1) which fits tanyas inverse graph. And shes right, the inverse of a function cannot be described by a single function.

Commentary

The student

- indicates that Tanya's graph is correct
- makes a conclusion based on only a single point from the graph, (–1, –2), which is insufficient to predict the entire graph

This response would receive a score of **2**

Scoring Criteria
- Tanya's graph is determined to be correct, and an attempt is made to provide a reason.

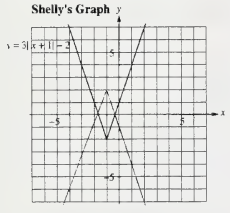
Student Task

Written Response – 4 marks

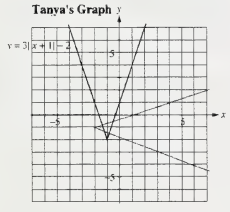
1. Shelly and Tanya are investigating the absolute value function. Both Shelly and Tanya correctly graph $y = 3|x + 1| - 2$, and then they each graph what they believe to be the inverse of this function.

   Either Shelly or Tanya constructed a correct graph. Shelly concludes that the inverse of this function is a function, whereas Tanya concludes that the inverse of this function cannot be described by a single function. Identify who is correct, and explain how you reached your conclusion.

Shelly's Graph



Tanya's Graph



Tanya's Graph is correct because the inverse of a function is never a function. To make it a function you would have to isolate the x value. Also the function is not ~~a~~ negative so the inverse would not be negative either.

Commentary

The student
- determines that Tanya's graph is correct
- states incorrectly that "the inverse of a function is never a function"; the rest of the statements are not useful in finding a solution

This response would receive a score of **1**

Scoring Criteria

- The student demonstrates some understanding of an inverse. For example, students make a statement like "interchange x and y."

OR

- Shelly's graph is determined to be correct. The reasoning includes true statements about Shelly's graph.

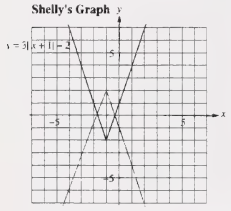
Student Task

Written Response – 4 marks

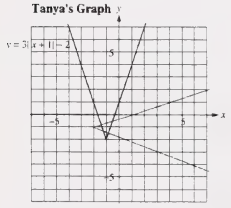
1. Shelly and Tanya are investigating the absolute value function. Both Shelly and Tanya correctly graph $y = 3|x + 1| - 2$, and then they each graph what they believe to be the inverse of this function.

Either Shelly or Tanya constructed a correct graph. Shelly concludes that the inverse of this function is a function, whereas Tanya concludes that the inverse of this function cannot be described by a single function. Identify who is correct, and explain how you reached your conclusion.

Shelly's Graph



Tanya's Graph



$y = 3(x+1) - 2$
$y = 3(x-1) + 2$

Shellys graph is correct because when you inverse the values you switch signs so $x = -1$ and $y = +2$ and therfore shellys inverse is correct and not Tanyas

Commentary

The student

- determines that Shelly's graph is correct
- provides an ordered pair, $(-1, -2)$, that is a coordinate on Shelly's graph

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Written Response 2: Student Task

Written Response – 4 marks

2. Most linear measurements in Canada are represented by a metric unit such as the metre. However, Canadian football continues to measure in yards. While watching a football game, Justin views an opening kick. The path that the football travels is parabolic, reaching a maximum height of 12 yards. When the football lands, it has travelled 60 yards down the field.

- Sketch the path that the football travels on a coordinate system. Assume that the origin represents the place where the football is kicked. Points plotted on the coordinate system should be of the form

  (distance football travels downfield, height of football).

- Identify the position of the horizontal intercepts and the vertex, and label the graph.

Scoring Criteria for Written Response 2

2 marks The student draws the sketch of a parabola that correctly models all the given information and in some way identifies the vertex, the horizontal intercepts, and labels the axes.

1 mark The student draws the sketch of a parabola that correctly models all the given information but fails to label the sketch correctly.

0 mark The student draws a sketch that does not model the information or shows no graph.

- Determine the equation of the relation between the height of the ball and its distance down the field from the kickoff point.

A Solution for Written Response 2

$y = -\frac{1}{75}(x-30)^2 + 12$ where x is the distance the football travels in yards and y is the height reached by the football.

Scoring Criteria for Written Response 2

2 marks The student obtains the equation $y = -\frac{1}{75}(x-30)^2 + 12$ or an equivalent form, or an equation that corresponds to the student's graph.

1 mark The student demonstrates some understanding of the relationship between the graph and the equation.

0 mark The student demonstrates little or no understanding of the relationship between the graph and the equation.

Written Response 2: Student Responses

This response would receive a score of **4**

Scoring Criteria

2 marks The student draws the sketch of a parabola that correctly models all the given information and in some way identifies the vertex, the horizontal intercepts, and labels the axes.

2 marks The student obtains the equation $y = -\frac{1}{75}(x-30)^2 + 12$ or an equivalent form, or an equation that corresponds to the student's graph.

Written Response – 4 marks

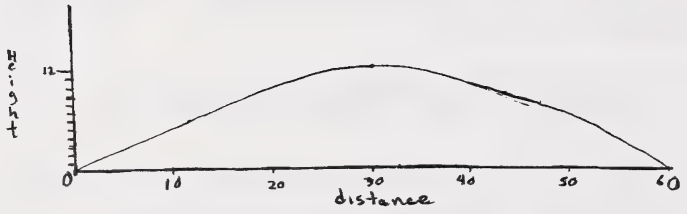
2. Most linear measurements in Canada are represented by a metric unit such as the metre. However, Canadian football continues to measure in yards. While watching a football game, Justin views an opening kick. The path that the football travels is parabolic, reaching a maximum height of 12 yards. When the football lands, it has travelled 60 yards down the field.

- Sketch the path that the football travels on a coordinate system. Assume that the origin represents the place where the football is kicked. Points plotted on the coordinate system should be of the form

(distance football travels downfield, height of football).

Identify the position of the horizontal intercepts and the vertex, and label the graph.

Hori zontal intercepts-(0,0) (60,0)
Vertex- (30,12)



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Continued

- Determine the equation of the relation between the height of the ball and its distance down the field from the kickoff point.

$h = -\frac{1}{75}(d-30)^2+12$

how I got the answer

It is a parabola so $y=a(x-h)+q$ or $h=a(d-h)^2+q$

h is the x value of the vertex and q is the y value so

$h=a(d-30)^2+12$

~~a=1/75~~

a can be determined by substituting a point on the graph's x and y values into the equation.

$0=a(0-30)^2+12$

$-12=a900$

$a=-\frac{12}{900}$

$a=-\frac{1}{75}$ so

$h=-\frac{1}{75}(d-30)^2+12$

Commentary

The student

- shows the sketch of a graph that correctly models the given information, labels the axes, and identifies the horizontal intercepts and the vertex
- correctly shows how an equation that correctly describes the given information is obtained

This response would receive a score of **3**

Scoring Criteria

2 marks The student draws the sketch of a parabola that correctly models all the given information and in some way identifies the vertex, the horizontal intercepts, and labels the axes.

1 mark The student demonstrates some understanding of the relationship between the graph and the equation.

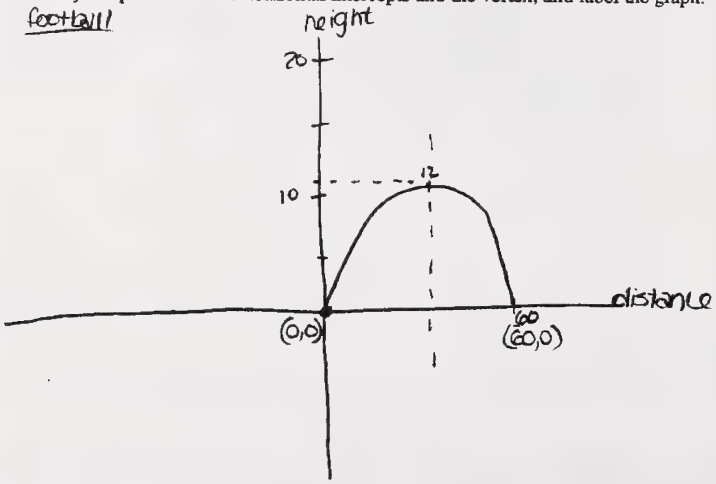
Written Response – 4 marks

2. Most linear measurements in Canada are represented by a metric unit such as the metre. However, Canadian football continues to measure in yards. While watching a football game, Justin views an opening kick. The path that the football travels is parabolic, reaching a maximum height of 12 yards. When the football lands, it has travelled 60 yards down the field.

- Sketch the path that the football travels on a coordinate system. Assume that the origin represents the place where the football is kicked. Points plotted on the coordinate system should be of the form

  (distance football travels downfield, height of football).

Identify the position of the horizontal intercepts and the vertex, and label the graph.



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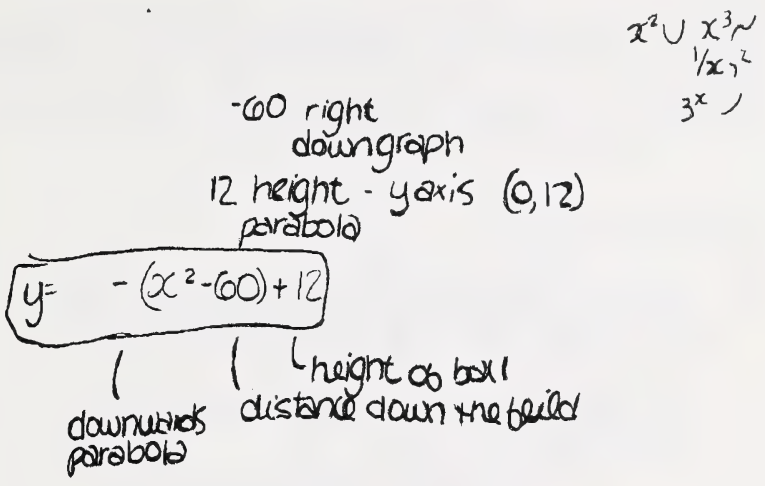
Continued

- Determine the equation of the relation between the height of the ball and its distance down the field from the kickoff point.



Commentary

The student

- correctly models the given information by drawing a parabola, and has properly labelled the axes and identified the vertex and horizontal intercepts
- correctly relates vertical shift and direction of opening of the graph to an equation
- incorrectly relates horizontal shift and vertical compression to an equation

This response would receive a score of **2**

Scoring Criteria

2 marks The student draws the sketch of a parabola that correctly models all the given information and in some way identifies the vertex, the horizontal intercepts, and labels the axes.

0 mark The student demonstrates little or no understanding of the relationship between the graph and the equation.

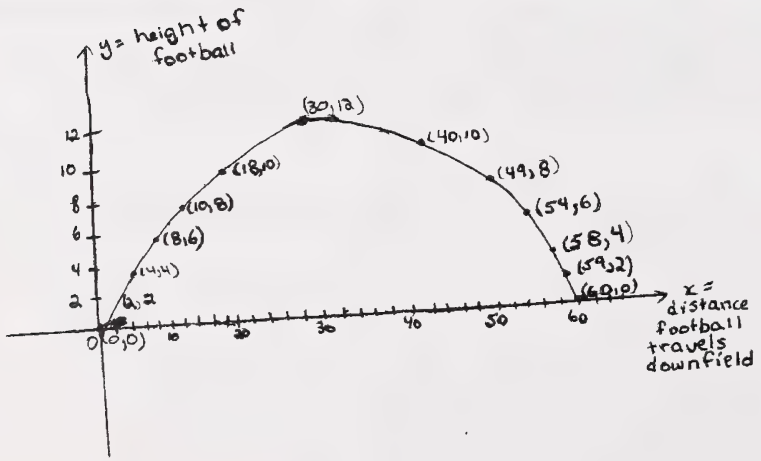
Written Response – 4 marks

2. Most linear measurements in Canada are represented by a metric unit such as the metre. However, Canadian football continues to measure in yards. While watching a football game, Justin views an opening kick. The path that the football travels is parabolic, reaching a maximum height of 12 yards. When the football lands, it has travelled 60 yards down the field.

- Sketch the path that the football travels on a coordinate system. Assume that the origin represents the place where the football is kicked. Points plotted on the coordinate system should be of the form

  (distance football travels downfield, height of football).

Identify the position of the horizontal intercepts and the vertex, and label the graph.



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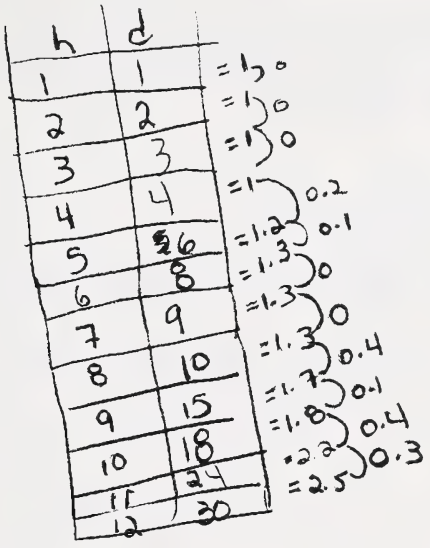
Continued

- Determine the equation of the relation between the height of the ball and its distance down the field from the kickoff point.



$h = 2.5(d)$

Commentary

The student

- illustrates all the given information from the problem in a diagram that shows a labelled axis, horizontal intercepts at $(0, 0)$ and $(60, 0)$, and a labelled vertex at $(30, 12)$
- incorrectly labels all other coordinates
- fails to demonstrate any understanding of the relationship between the graph and an equation

This response would receive a score of **1**

Scoring Criteria

1 mark The student draws the sketch of a parabola that correctly models all the given information but fails to label the sketch correctly.

0 mark The student demonstrates little or no understanding of the relationship between the graph and the equation.

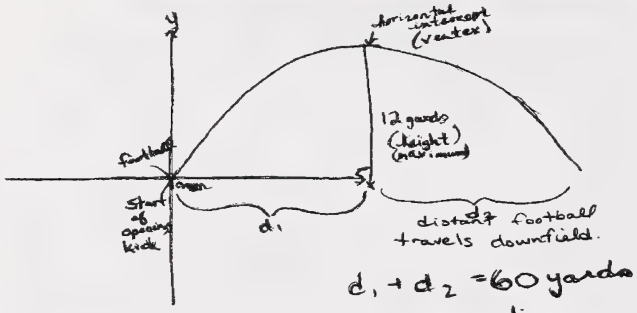
Written Response – 4 marks

2. Most linear measurements in Canada are represented by a metric unit such as the metre. However, Canadian football continues to measure in yards. While watching a football game, Justin views an opening kick. The path that the football travels is parabolic, reaching a maximum height of 12 yards. When the football lands, it has travelled 60 yards down the field.

- Sketch the path that the football travels on a coordinate system. Assume that the origin represents the place where the football is kicked. Points plotted on the coordinate system should be of the form

(distance football travels downfield, height of football).

Identify the position of the horizontal intercepts and the vertex, and label the graph.



$d_1 + d_2 = 60$ yards

$60 - 12 = d_1$

48

The ball travelled 48 yards before reaching it's maximum height of 12 yards. The ball continued to travel another 12 yards before landing on the ground.

Continued

Continued

- Determine the equation of the relation between the height of the ball and its distance down the field from the kickoff point.

The maximum height of 12 yards is equal to the remaining distance that the ball travelled after that point before reaching the ground

The ball travelled 48 yards before reaching it's maximum height of 12 yards.

You can find this by taking the balls total distance travelled, 60 yard and subtract the maximum height reached which gives you the first distance travelled which is 48 yards to find the remaining distance you take total yards minus the first distance. (60 - 48 = 12) which then gives you 12 yards.

Commentary

The student

- correctly models the information on the graph but inappropriately labels the axes
- shows a lack of any understanding of the relationship between the graph and an equation by determining the distance travelled by the ball is determined by subtracting the maximum height reached, 48 yards, from the total distance travelled, 60 yards

Mathematics 20

Performance Assessment

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MATERIALS

The Student Responses for Performance Assessment follow on page 22.

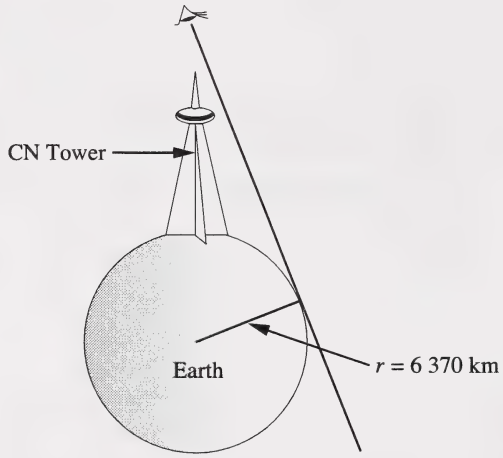
Performance Assessment: Student Task

According to the song, *"on a clear day you can see forever"*; however, on a curved planet this is **not** true!

The following tasks are concerned with the range of vision of an observer on a curved planet.

In order to assist you in modelling parts of the following tasks, obtain from your teacher:
- 1 ice cream pail lid
- a needle
- a piece of thread or string

Suppose that from the top of the CN Tower which is 400 m high, your line of vision is as shown below. The Earth has a radius of 6 370 km.



- Calculate the farthest distance that can be seen from the top of the CN Tower. A model simulating this situation may assist you with your solution.
- Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?
- Derive a formula expressing the maximum distance seen (d) in terms of the radius of a planet (R) and the height of the observer (h).

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Scoring Criteria for Performance Assessment	
5 marks	The student • demonstrates an understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find the solution • justifies the solution
4 marks	The student • demonstrates an understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find a solution, but the solution contains a minor flaw • shows some justification for results
3 marks	The student • demonstrates some understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find *partial* solutions • communicates little understanding of the complexities of the problem • formulates some aspects of the problem mathematically
2 marks	The student • explores the initial stages of the problem • applies some relevant mathematical knowledge and problem-solving techniques to find *partial* solutions
1 mark	The student • applies some relevant mathematical knowledge to the problem
0 mark	The student • is off topic, or leaves the paper blank

Performance Assessment: Student Responses

This response would receive a score of **5**

Scoring Criteria

The student
- demonstrates an understanding of the problem
- uses appropriate mathematical knowledge and problem-solving techniques to find the solution
- justifies the solution

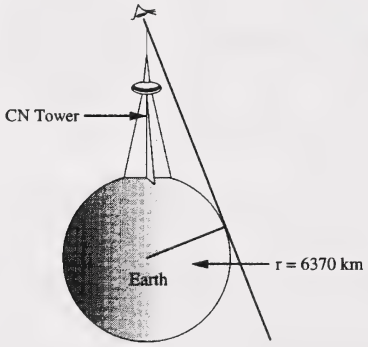
Student Task

According to the song, *"on a clear day you can see forever"*; however, on a curved planet this is **not** true!

The following tasks are concerned with the range of vision of an observer on a curved planet.

In order to assist you in modelling parts of the following tasks, obtain from your teacher:
- 1 ice cream pail lid
- a needle
- a piece of thread or string

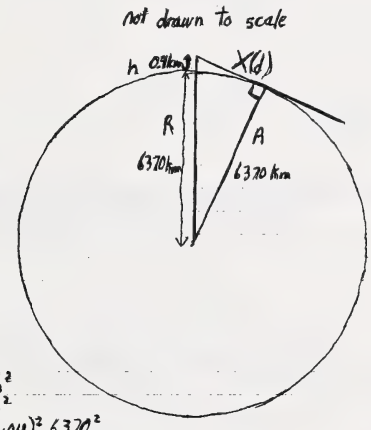
Suppose that from the top of the CN Tower which is 400 m high, your line of vision is as shown below. The Earth has a radius of 6 370 km.



Continued

- Calculate the farthest distance that can be seen from the top of the CN Tower. A model simulating this situation may assist you with your solution.
- Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?
- Derive a formula expressing the maximum distance seen (d) in terms of the radius of a planet (R) and the height of the observer (h).

Student Task



1. $h^2=a^2+b^2$
$a^2=h^2-b^2$
$a^2=(6370+0.1)^2-6370^2$
$\sqrt{a^2}=\sqrt{5096.16}$
$a=71.4$ km is the distance you can see from the top of the N.tower

2. $d=\sqrt{a^2+b^2}-R$
$d=\sqrt{71.4^2+6370^2}-6370$
$d=\sqrt{5097696.16}-6370$
$d=6370.001661-6370$
$d=0.001660961$ km $=1.7$m

3. $h^2=a^2+b^2$
$a^2=h^2-b^2$
$d^2=(R+h)^2-R^2$
$d=\sqrt{(R+h)^2-R^2}$

Commentary

The student

- demonstrates a good knowledge and understanding of the problem by applying the radius–tangent theorem
- uses appropriate problem-solving techniques to find the solution by applying Pythagorean Theorem
- justifies the solution algebraically

This response would receive a score of **4**

Scoring Criteria
The student
- demonstrates an understanding of the problem
- uses appropriate mathematical knowledge and problem-solving techniques to find a solution, but the solution contains a minor flaw
- shows some justification for results

Student Task

According to the song, *"on a clear day you can see forever"*; however, on a curved planet this is **not** true!

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In order to assist you in modelling parts of the following tasks, obtain from your teacher:
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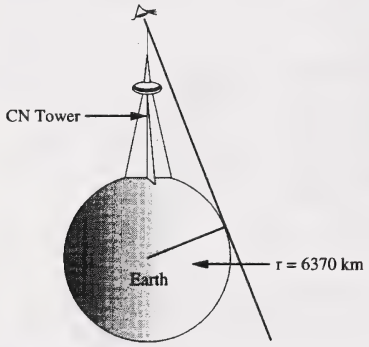
- a piece of thread or string

Suppose that from the top of the CN Tower which is 400 m high, your line of vision is as shown below. The Earth has a radius of 6 370 km.



Continued

Continued

1000m = 1km
400m = 0.4km

- Calculate the farthest distance that can be seen from the top of the CN Tower. A model simulating this situation may assist you with your solution.

$x^2 + 6370^2 = 6370.4^2$
$x^2 = 5096.16$
$x = 71.4\ km$

1000 m = 1km
0.001661km

- Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?

$4.6^2 + 6370^2 = x^2$
$21.16 + 40576900 = x^2$
$x^2 = 40576921.16$
$x = 6370.001661$
$6370.001661 - 6370 = 0.001661\ km$ or

1.661 m

- Derive a formula expressing the maximum distance seen (d) in terms of the radius of a planet (R) and the height of the observer (h).

$d^2 + R^2 = (R+h)^2$

Commentary

The student
- shows a good knowledge and understanding of the problem by applying the tangent–radius theorem
- correctly solves the first two parts of the problem, showing all work, including units
- makes a minor error in Part 3, where d is not isolated as required in the question

This response would receive a score of **3**

Scoring Criteria
The student
- demonstrates some understanding of the problem
- uses appropriate mathematical knowledge and problem-solving techniques to find *partial* solutions
- communicates little understanding of the complexities of the problem
- formulates some aspects of the problem mathematically

Student Task

According to the song, *"on a clear day you can see forever"*; however, on a curved planet this is **not** true!

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In order to assist you in modelling parts of the following tasks, obtain from your teacher:
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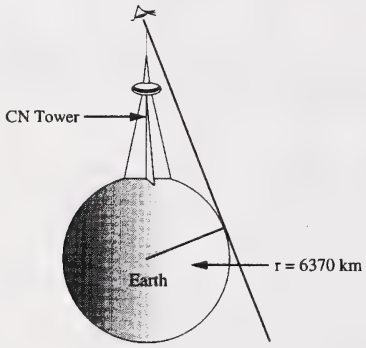
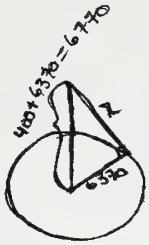


Continued

Continued

- **Calculate the farthest distance that can be seen from the top of the CN Tower. A model simulating this situation may assist you with your solution.**



$6770^2 = x^2 + 6370^2$

$45832900 = x^2 + 40576900$

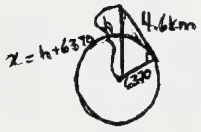
$5256000 = x^2$

$2292.6 = x$

$= 2292.6 \text{ km}$

Student Task (continued)

- **Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?**



$x^2 = 4.6^2 + 6370^2$

$= 21.16 + 40576900$

$x^2 = 40576921$

$x = 6370 \text{ km} + h$

$h = 6370.0017 - 6370$

$= 0.0016609 \text{ km}$

$= 1.66 \text{ m}$

- **Derive a formula expressing the maximum distance seen (d) in terms of the radius of a planet (R) and the height of the observer (h).**

$d = 4.6 \text{ m}$

$R = 6370$

$h = 1.66 \text{m}$

$d = \sqrt{2Rh}$

Commentary

The student

- demonstrates some understanding of the problem by showing a tangent meeting the radius at a right angle
- uses appropriate mathematical knowledge by applying Pythagorean Theorem in Part 1, but fails to convert units, thus obtaining 6 770 instead of 6 370.4 which results in a wrong answer for Part 1
- provides a correct solution for Part 2
- gives an invalid response for Part 3

This response would receive a score of **2**

Scoring Criteria
The student
- explores the initial stages of the problem
- applies some relevant mathematical knowledge and problem-solving techniques to find *partial* solutions

Student Task

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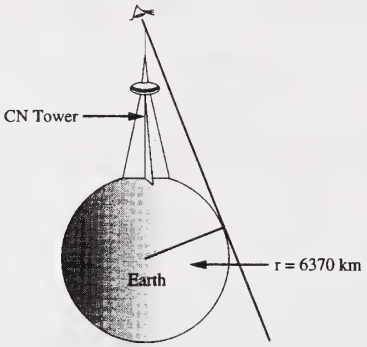

Continued

Continued

- Calculate the farthest distance that can be seen from the top of the CN Tower.
  A model simulating this situation may assist you with your solution.

$h = 6370.4$
$b = 6370$
$a = ?$

$h^2 = a^2 + b^2$
$(6370.4)^2 = a^2 + 6370^2$
$40581996.16 = a^2 + 40576900$
$\sqrt{a^2} = \sqrt{5096.16}$
$a = 71.39$ km

- Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?

- Derive a formula expressing the maximum distance seen (d) in terms of the radius of a planet (R) and the height of the observer (h).

Commentary

The student
- explores the initial stages of the problem by showing a good understanding of the Pythagorean Theorem and obtaining a correct solution to the first part
- provides no solution for the second and third parts

This response would receive a score of **1**

Scoring Criteria
The student
- applies some relevant mathematical knowledge to the problem

Student Task

According to the song, *"on a clear day you can see forever"*; however, on a curved planet this is **not** true!

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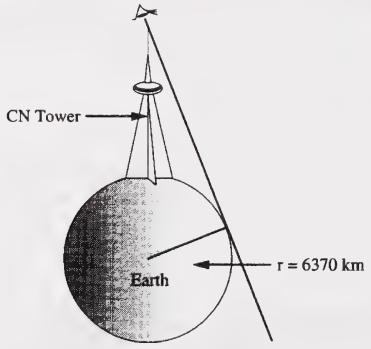


Continued

Continued

- **Calculate the farthest distance that can be seen from the top of the CN Tower. A model simulating this situation may assist you with your solution.**

$d = \sqrt{2Rh}$

$= \sqrt{2(6370\text{km})(0.553\text{km})}$

$= \boxed{83.9\text{ km}}$

- **Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?**

- Derive a formula expressing the maximum distance seen (d) in terms of the radius of a planet (R) and the height of the observer (h).

$d^2 = (h+r)^2 + r^2$

$d = \sqrt{(h+r)^2 + r^2}$

Commentary

The student
- demonstrates knowledge of the Pythagorean theorem to solve the third problem, but incorrectly positions the right-angle

Mathematics 20

Projects A & B

CLASSROOM
ASSESSMENT
MATERIALS

Project A: Student Task

Your project must be based upon the theme *Relations and Functions*. However, you must discuss your choice of topic and how it relates to the theme with your teacher. The examples listed below are **possible starting points.** It is possible to combine related ideas, but keep in mind that an investigative project is intended to include **depth** as well as breadth of research. Choose one of the examples below or a related topic of your own, for your project.

Using a recognized computer program or a graphing calculator

1. The order in which operations are performed in mathematics can be important. Investigate the effects of varying the order of introducing changes in the values of the parameters (one at a time) in the graphs of $y = cf(x - a) + b$.
2. During your Mathematics 20 course, you have studied functions such as linear, quadratic, reciprocal, cubic, exponential, and absolute value. Investigate a minimum of two other functions to determine if the properties of transformations are consistent with what you have learned. As an extension, you may wish to investigate transformations applied to relations such as circles.
3. Investigate how parabolas are used in everyday applications such as suspension bridges (e.g., the Golden Gate Bridge), satellite dishes, arches, car headlights, etc.
4. Many phenomena in physics and chemistry are described by equations involving variation (e.g., Charles' Law and Boyle's Law in chemistry, etc). Investigate a minimum of three such equations, identifying which are direct, inverse, or partial variations, the domain and range, and the dependent and independent variables. Using a recognized computer graphing package, on the same set of axes produce graphs from which a variety of information can be read. For example, on one set of axis, show the relationship of temperature and volume for Charles' Law, producing several graphs that each show this relationship for a different fixed value of pressure.
5. Using a minimum of three types of functions studied in Mathematics 20 (e.g., parabolas, exponential, absolute value), create a computer program that displays a kaleidoscope of colours. An alternative would be to create a static two-dimensional or three-dimensional design.

In your investigative project:

- include a statement with supporting arguments, as to whether the relations you have worked with are functions
- determine all domains and ranges
- describe, explain, and illustrate, where appropriate, any transformations; i.e., reflections, vertical stretches, translations
- determine the equations of the functions used
- identify x-intercepts of graphs, zeros of functions, and roots of equations, as appropriate
- acknowledge which areas of mathematics you use: arithmetic, geometry, trigonometry, and/or algebra
- include the type of computer software program or graphing calculator you used and your own analysis

Project B: Student Task

It has often been said that mathematics is a subject that loses a great deal by not rooting it in its history. The subject is taught and learned with little or no reference to the people who contributed major ideas and who were responsible for the advancement of mathematical thought.

Your project must be based on the theme ***Math history: a neglected domain***. Select one of the topics listed below, or, if your prefer, investigate any topic related to this theme. You must however, discuss your choice of topic with your teacher.

Projects should reflect **depth** as well as **breadth** of research. It is expected that your report will include a thorough but concise biography of the mathematicians named in each question (this may be presented in the form of a poster), a precise discussion of the individual's contribution to the advancement of mathematics, and references to where these ideas are reflected in our mathematics curriculum. Choose one of the examples below or a related topic of your own, for your project.

1. **John Napier** is credited with the invention of logarithms and a tool for calculation called Napier's Rods or Napier's Bones. Provide a complete and clear explanation of both of these inventions.
2. **Blaise Pascal** is recognized for the introduction and use of Pascal's triangle. In fact, Pascal did not develop Pascal's triangle. Find the origin of this famous triangle, and provide a number of examples of it use.
3. **Sonya Kovalevski, Sophie Germain,** and **Emmy Noether** are three women mathematicians who had the courage and determination to enter a male-dominated field, and contribute to the development of mathematics. Discuss their contributions and provide explanations of the social and cultural obstacles that may have prevented women from becoming mathematicians. Describe the environment that a woman would face today if she elected to specialize in mathematics.
4. **George Pólya**, a twentieth-century mathematician and author of *How to Solve It*, is considered a pioneer in the field of problem solving. Discuss his approach to problem solving and explain if, in your view, his methods are reflected in the way mathematics is taught in our schools. Select one of his famous problems and provide a clear and logical solution.
5. **René Descartes** is the inventor of analytical geometry. Give a description of this branch of mathematics and compare it to Euclidean geometry. Explain the importance of analytic geometry in the development of mathematics. Give a description of a circle using Euclidean geometry, Cartesian geometry and algebra.
6. **Sir Isaac Newton** and **Gottfried Wilhelm Leibniz** are considered rivals in mathematics. Provide examples of this rivalry and focus on what became known as the Newton-Leibniz controversy.

Mathematics 20

Scoring Criteria for Project					
Scoring Categories	**Not Shown** ( ) marks	**Low** ( ) marks	**Medium** ( ) marks	**High** ( ) marks	**Comments**
Defining the Project (10%)	(0)	(1)	(2)	(3)	
Conducting the Project (20%)	(0)	(2)	(4)	(6)	
Analysis (20%)	(0)	(2)	(4)	(6)	
Mathematical Content (30%)	(0)	(3)	(6)	(9)	
Communication (20%)	(0)	(2)	(4)	(6)	

Total marks: (30) _____%

Using the Scoring Criteria

A student's project will be rated on several categories; and the scores summed from each category to obtain a total mark for the entire project. This type of scoring, referred to as *anaholistic*, provides more information about a task and furnishes information from different perspectives, pinpointing strengths and weaknesses. The *Scoring Criteria* has three categories that relate specifically to parts of the project. These are *Defining the Project, Conducting the Project,* and *Analysis*. The last two categories, *Mathematical Content* and *Communication*, are intended to be used as overall assessment categories for the entire project.

In the body of the table, possible marks for each category are indicated in brackets. For example, the total possible marks for the category *Defining the Project* is 3, whereas the total possible marks for *Mathematical Content* is 9. Each category has a continuum of marks. For example, a student may receive a mark anywhere from 0 to 9 on the *Mathematical Content*. The marks allotted in each of the five sections of categories are summed for a possible total of 30.

The detailed categories for the *Scoring Criteria* are on the next page.

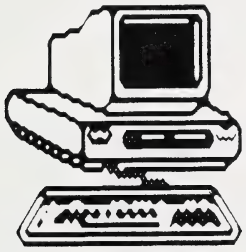
Scoring Categories for Mathematics 20: Projects A and B	*Ratings*
Defining the Project: • identifying information required • asking useful questions • acknowledging difficulties and limitations • suggesting extensions • statement of project topic • statement of purpose of project	3 – thoroughly evident 2 – substantially evident 1 – minimally evident 0 – not shown
Conducting the Project: • collecting appropriate factual information • consulting appropriate resources (peers, teachers, references) • organizational skills	6 – exemplary 4 – effective 2 – marginal 0 – not evident
Analysis: • analyzing information • interpreting and critically evaluating the mathematicians' contribution to mathematics • drawing logical conclusions • evaluation of conclusions	6 – thoroughly developed 4 – partially developed 2 – minimally developed 0 – not developed
Overall Categories	***Ratings***
Mathematical Content: • connection of the mathematics researched to our curriculum • use of mathematical language and symbols • mathematical formulation or interpretation of problem • interpretation and evaluation of mathematics used	9 – proficient 6 – adequate 3 – marginal 0 – unsatisfactory
Communication: • coherent presentation • information clearly presented • references cited • acknowledgment of assistance • appendices	6 – clear, well-organized, complete 4 – mostly clear and well-organized 2 – some clear parts 0 – unclear and disorganized

Project A: Student Response

This response would receive a score of **26**

Math 20 Project

A Pattern of Functions



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"Do not worry about your difficulties in Mathematics, I can assure you, mine are still greater."
Albert Einstein

Statement of Topic
Relations and Functions. These are the tools which I used in creating a kaleidoscope, with an intricate, yet recognizable design once broken down into it's basic functions. On the computer, graphs are easy to manipulate and can create patterns, which amuse, yet show us just one more way mathematics is used, and can be used and seen in everyday life.

-3-

All the relations I worked with were functions, this is true, and can be proven because no 'x' value has any more then one 'y' value, and the vertical line test proves it as well because when a vertical line is placed anywhere on the graph it only crosses the graph at one location.

I chose three functions and their negatives for this investigative report, $y = 50(1/x)$, $y = 50(-1/x)$, $y = ABS(x)$, $y = -(ABS(x))$, $y = x^2/4 + 25$, $y = -((x^2/4) + 25)$.

The domain and ranges for each of the functions is as follows:

Absolute:
(+)
Domain: All Real Numbers
Range : $y \geq 0$
(-)
Domain: All Real Numbers
Range : $y \leq 0$

Reciprocal:
(+)
Domain: $x \neq 0$
Range : $y \neq 0$
(-)
Domain: $x \neq 0$
Range : $y \neq 0$

Quadratic:
(+)
Domain: All Real Numbers
Range : $y \geq -25$
(-)
Domain: All Real Numbers
Range : $y \leq 25$

After choosing the the functions I was going to use, I had to choose a programming language to code it in. I chose QBasic v.1.0 due to its simplicity (explanation later on).

-4-

Calling upon my extensive knowledge of QBasic, I was able to code it in about an hour and a half.

Continued

Continued

I used QBasic version 1.0 to program my Math project. I chose it over Turbo Pascal v.7.0 because of its simplicity for other people to understand. It was fast for me to program what I wanted. With QBasic 1.0 I could easily show the function equations so an average person could scan the source code and pick out equations.

With QBasic however, I had to change things around. First of all, on a computer, a pixel, or PIcture-X-ELement (little dot of colored light) is about .28 to .39mm big and so I had to change the graphs to show up nice. The computer screen is also nota perfect square and the screen mode I used, had a resolution of 320x200 pixels. So the scale for the x- and y-axes were not the same. On the screen as well, (0,0) are in the top left-hand corner. (The people that design computers don't really get out much I think...)

I used Microsoft Works 3.0 to type out my report, which is an all-in-one package that has a spreadsheet, database, and word processor all built together and integrated. It is fairly user-friendly, (not bad for Microsoft), and a nice cheap all-in-one package. I printed the source code out using a utility called FOLD-and-PRINT version 3.3, which makes it easy to look it over all in one page.

The computer I used, which is my home computer is a Packard Bell Legend II, a 80386 running at 12 MHz with 640kb of RAM, and a 40MB hard drive, with a VGA monitor.

The following section is just to get QBasic Initialized for me to work in so that I can use simple "y= " equations:

```
RANDOMIZE TIMER
pi = 3.1415 / 2
SCREEN 2: SCREEN 0
SCREEN 7
LINE (0, 100)-(319, 100), 8          'This is the X axis
LINE (160, 0)-(160, 199), 8          'This is the Y axis

start:
DO
'Randomize Colors for Each of the Graphs so it looks pretty
(g)
colr = INT(RND * 14) + 1
colr2 = INT(RND * 14) + 1
colr3 = INT(RND * 14) + 1
colr4 = INT(RND * 14) + 1
colr5 = INT(RND * 14) + 1
colr6 = INT(RND * 14) + 1
```

This is where the important part of the program starts. It repeats four times because the kaleidoscope has all of the graphs in the middle, and then one centered in each corner.

```
FOR theta = 0 TO 2 * pi STEP .005
x = COS(theta) * 100          'Define the 'X' Value, needed
because of QBASIC
```

This is where the equations take place
the PSET is QBasic for PenSET, or where the computer will leave a dot of light, thus plotting the function graph
All the formulas are in BOLD

```
y = (1 / x) * 50          ' Reciprocal Function
PSET (x + 160, y + 100), colr
y = -(1 / x) * 50         ' Negative Reciprocal Function
PSET (x + 160, y + 100), colr2
y = ABS(x)                ' Absolute Function
PSET (x + 160, y + 100), colr3
y = -(ABS(x))             ' Negative Absolute Function
PSET (x + 160, y + 100), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 160, y + 100), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 160, y + 100), colr6
```


This is where the program plots the four graphs that go in each corner of the kaleidoscope

```
y = (1 / x) * 50          ' Reciprocal function
PSET (x + 69, y + 50), colr
y = -(1 / x) * 50         ' Negative Reciprocal Function
PSET (x + 69, y + 50), colr2
y = ABS(x)                ' Absolute Function
PSET (x + 69, y + 50), colr3
y = -(ABS(x))             ' Negative Absolute Function
PSET (x + 69, y + 50), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 69, y + 50), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 69, y + 50), colr6


y = (1 / x) * 50          ' Reciprocal function
PSET (x + 250, y + 50), colr
y = -(1 / x) * 50         ' Negative Reciprocal Function
PSET (x + 250, y + 50), colr2
y = ABS(x)                ' Absolute Function
PSET (x + 250, y + 50), colr3
y = -(ABS(x))             ' Negative Absolute Function
PSET (x + 250, y + 50), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 250, y + 50), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 250, y + 50), colr6


y = (1 / x) * 50          ' Reciprocal function
PSET (x + 250, y + 150), colr
y = -(1 / x) * 50         ' Negative Reciprocal Function
PSET (x + 250, y + 150), colr2
y = ABS(x)                ' Absolute Function
PSET (x + 250, y + 150), colr3
y = -(ABS(x))             ' Negative Absolute Function
PSET (x + 250, y + 150), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 250, y + 150), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 250, y + 150), colr6
```



```
y = (1 / x) * 50          ' Reciprocal function
PSET (x + 69, y + 150), colr
y = -(1 / x) * 50         ' Negative Reciprocal Function
PSET (x + 69, y + 150), colr2
y = ABS(x)                ' Absolute Function
PSET (x + 69, y + 150), colr3
y = -(ABS(x))             ' Negative Absolute Function
PSET (x + 69, y + 150), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 69, y + 150), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 69, y + 150), colr6
```

This part just completes the loop and tells the computer to start over with different colors until someone presses a key

```
NEXT theta

LOOP WHILE INKEY$ = ""  'Press any key to Quit
```

In conclusion, I'd like to say this was a fun project and was enjoyed. Computers really help people do repetitive tasks faster and calculations way faster then humans.

Limitations encountered during the investigation was that my computer did things way too slow, and limited the amount of graphs that could be plotted without taking an eternity. Some possible further investigations that could take place because of this project are ones that deal with art, and the way functions and graphs are used in modern art, and the way people design pictures.

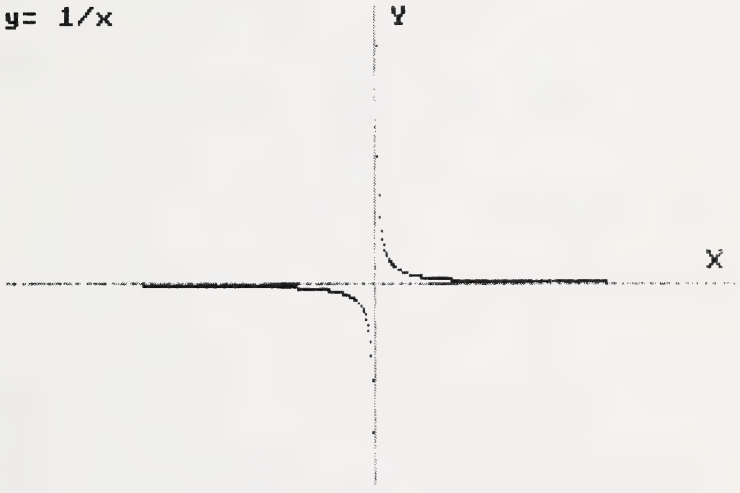
The kind of mathematical knowledge needed and encountered during this investigation was about functions and relations.

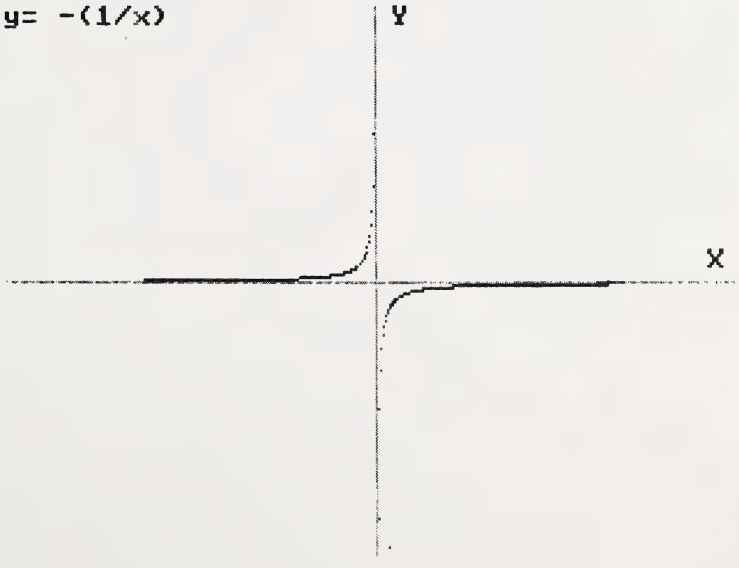
WORD COUNT: 1519

Continued

Continued

Appendice I
Equations and Graphs





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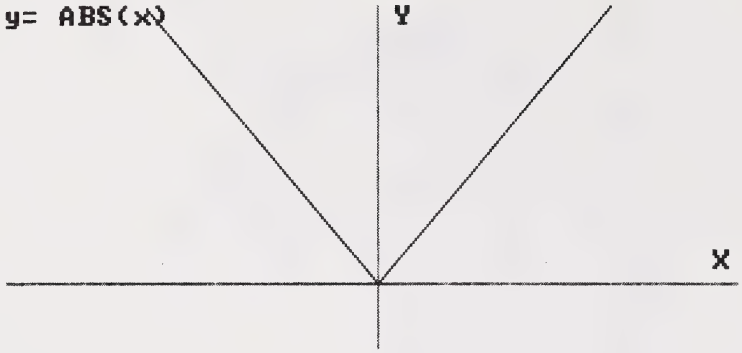

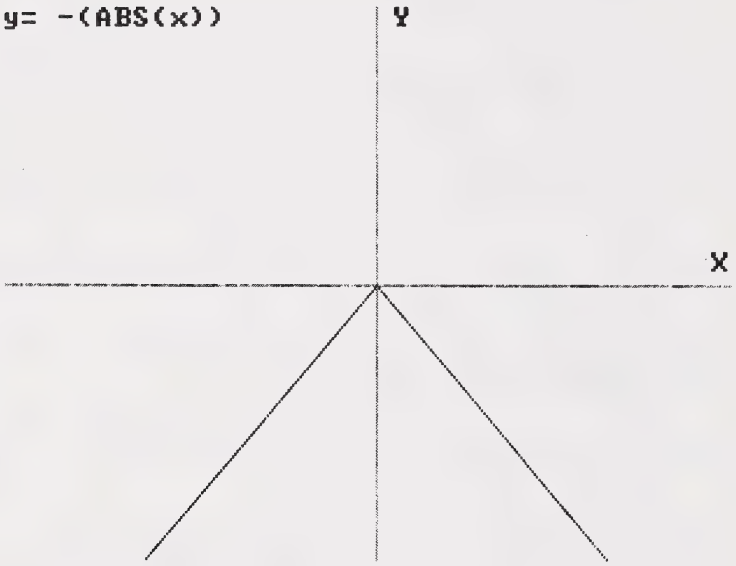

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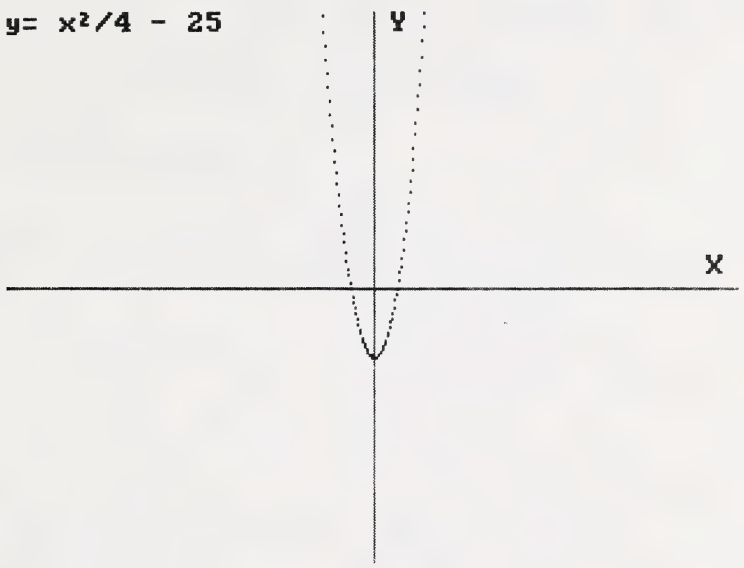

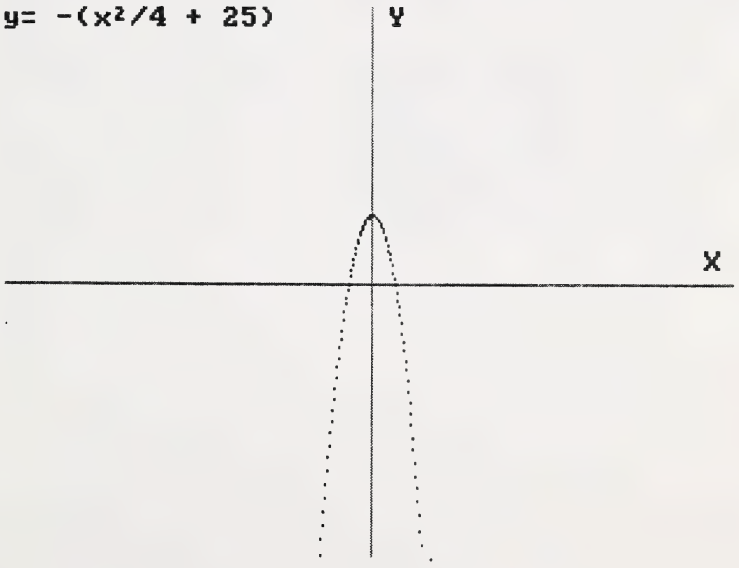

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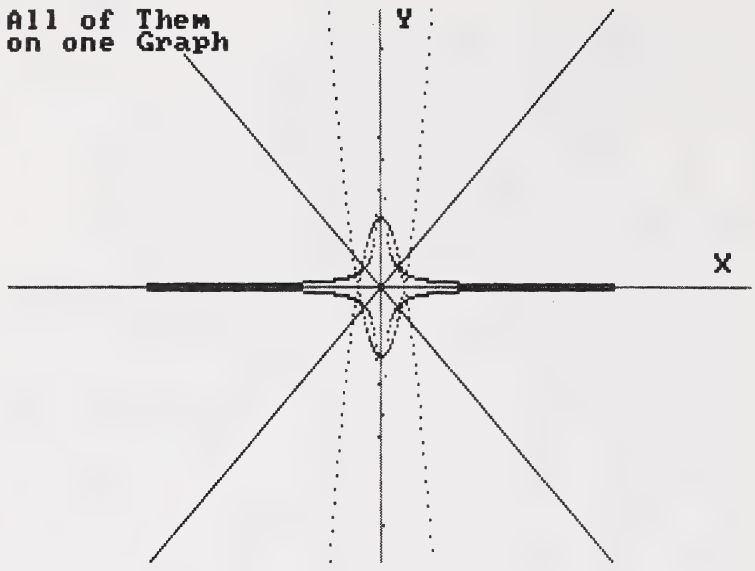

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Appendice II
Program Listing and Output

Printed using FOLD-and-PRINT ver. 3.3 on 12-13-1995, 12:16:43 AM project.bas

```
RANDOMIZE TIMER
pi = 3.1415 / 2
    W 2: SCREEN 0
    W 7
LINE (0, 100)-(319, 100), 8         'This is the X axis
LINE (160, 0)-(160, 199), 8         'This is the Y axis

start:
DO
'Randomize Colors for Each of the Graphs so it looks pretty (g)
colr = INT(RND * 14) + 1
colr2 = INT(RND * 14) + 1
colr3 = INT(RND * 14) + 1
colr4 = INT(RND * 14) + 1
colr5 = INT(RND * 14) + 1
colr6 = INT(RND * 14) + 1



FOR theta = 0 TO 2 * pi STEP .005
x = COS(theta) * 100          'Define the 'X' Value, needed because of QBASIC

y = (1 / x) * 50        ' Reciprocal Function
PSET (x + 160, y + 100), colr
y = -(1 / x) * 50       ' Negative Reciprocal Function
PSET (x + 160, y + 100), colr2
y = ABS(x)              ' Absolute Function
PSET (x + 160, y + 100), colr3
y = -(ABS(x))           ' Negative Absolute Function
PSET (x + 160, y + 100), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 160, y + 100), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 160, y + 100), colr6


    1 / x) * 50        ' Reciprocal Function
    (x + 69, y + 50), colr
y = -(1 / x) * 50       ' Negative Reciprocal Function
PSET (x + 69, y + 50), colr2
y = ABS(x)              ' Absolute Function
PSET (x + 69, y + 50), colr3
y = -(ABS(x))           ' Negative Absolute Function
PSET (x + 69, y + 50), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 69, y + 50), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 69, y + 50), colr6

y = (1 / x) * 50        ' Reciprocal Function
PSET (x + 250, y + 50), colr
y = -(1 / x) * 50       ' Negative Reciprocal Function
PSET (x + 250, y + 50), colr2
y = ABS(x)              ' Absolute Function
PSET (x + 250, y + 50), colr3
y = -(ABS(x))           ' Negative Absolute Function
PSET (x + 250, y + 50), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 250, y + 50), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 250, y + 50), colr6


y = (1 / x) * 50        ' Reciprocal Function
PSET (x + 250, y + 150), colr
y = -(1 / x) * 50       ' Negative Reciprocal Function
PSET (x + 250, y + 150), colr2
y = ABS(x)              ' Absolute Function
PSET (x + 250, y + 150), colr3
y = -(ABS(x))           ' Negative Absolute Function
PSET (x + 250, y + 150), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 250, y + 150), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 250, y + 150), colr6


y = (1 / x) * 50        ' Reciprocal Function
PSET (x + 69, y + 150), colr
y = -(1 / x) * 50       ' Negative Reciprocal Function
PSET (x + 69, y + 150), colr2
y = ABS(x)              ' Absolute Function
PSET (x + 69, y + 150), colr3
y = -(ABS(x))           ' Negative Absolute Function
PSET (x + 69, y + 150), colr4
y = ((x ^ 2) / 4) - 25  ' Quadratic Function
PSET (x + 69, y + 150), colr5
y = (-(x ^ 2) / 4) + 25 ' Negative Quadratic Function
PSET (x + 69, y + 150), colr6

NEXT theta

LOOP WHILE INKEY$ = ""  'Press any key to Quit
```

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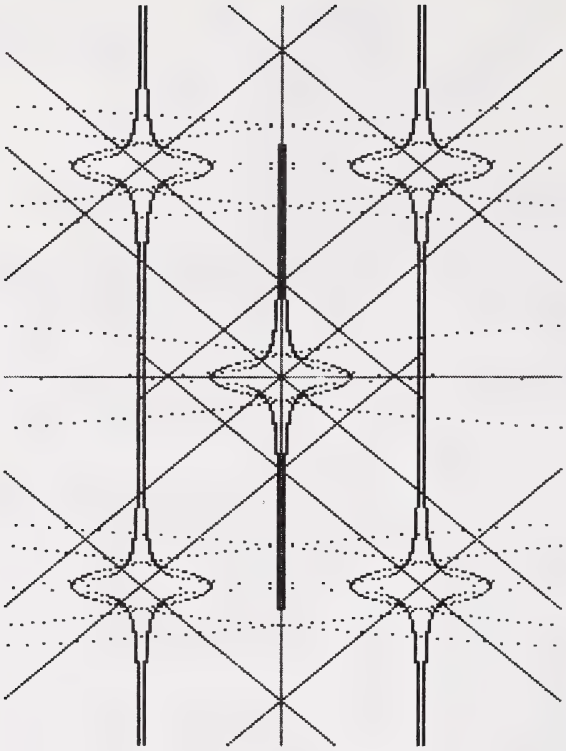


Continued

Commentary *This response would receive a score of* **26**

Defining the Project (3)
The student
- states the project topic and clearly identifies information required
- addresses the difficulties of working with *QBasic*

Conducting the Project (6)
The student
- clearly presents appropriate data
- organizes the information well

Analysis (5)
The student
- did not address horizontal or vertical translation, stretch, or compression

Mathematical Content (8)
The student
- misses out some relevant mathematical information, such as reference to zeros of the functions and roots of the equation

Communication (4)
The student
- makes a coherent presentation
- includes program listing in the body of the report and the appendix

Project B: Student Response

This response would receive a score of **25**

Math History: A Neglected Domain
John Napier

TABLE OF CONTENTS

I. MAIN TEXT
a)Introduction
b)History of John Napier
c)Napier's Bones
i)Diagram
d)Use of Napier's Bones
e)Logarithms
f)Use of logarithms
g)Conclusion

II. SUMMARY SECTION
a)Project summary
b)Limitations
c)Encountered math knowledge

III. REFERENCES

Continued

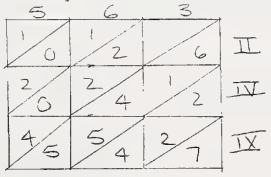
Continued

I MAIN TEXT

The mathematician, John Napier, is the man credited with the inventions of logarithms and Napier's bones, a calculation tool. The history of this man and his inventions, although not often thought of, or perhaps even forgotten by most, affect our lives unknowingly everyday. The math taught today loses a great deal in not learning who contributed the time and effort for the major theories that made our advanced mathematical position possible. Without the discoveries made by John Napier, the time and concentration spent on the little things, like multiplication, would take many, many, times longer than it does today.

To begin, the Scottish mathematician, John Napier, son of Archibald Napier, was born in 1550, at Merchiston Castle. He attended St. Andrews University at the age of thirteen, but only for a brief period of time, and married in 1571. The Eighth Laird of Merchiston, Napier spent the majority of his life campaigning against Roman Catholic supremacy. He was a loyal presbyterian who wrote the book, A Plaine Discoverie of the Whole Revelation of Saint John, identifying the Pope as the Antichrist. As a result, much of Napier's time was spent campaigning, rather than on mathematics. In fact, his calculations for the book on logarithms alone took twenty years. Napier was the first to use the decimal point in writing fractions. Most of Napier's works was done in Latin, in which he was more fluent than he was in English. John Napier died on April 4, 1617.

In 1617, Napier wrote the book Rabdologiae, describing his famous `bones', or `rods', as he called them. The `bones', a calculating device used to multiply and divide numbers, can be made from paper or wood, cut into 10 strips, 5 inches long, and 1/2 inch wide. The strips are written on and arranged like the table on the following page. The strips make up a multiplication table, or if used in reverse, a division table. To multiply single digits, like 4*5, one must look under the '4' column to the row, labelled Roman numeral V (5). The answer is 2/0 or 20. For more complex questions like 563*249, the concept is slightly more difficult, but once grasped, it is an easy way to calculate. First, the strips, or `bones', 5, 6, and 3 are removed from the table, as well as the marker containing all the Roman numerals on the right hand side, like so:



To begin, we break down 563 multiplying by 9 units, 4 tens, and 2 hundreds, (since 249 = 200 + 40 + 9), as shown:

561 * 249
9 * 563 =
40 * 563 =
200 * 563 =
249 * 563

To multiply 9 * 563, look along line IX of the Roman marker, for the last digit of 563: 3. We see that nine threes is 27, thus we use the seven as the last digit in `9 * 563 = ' (above), and

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1) Diagram

Napier's Bones: Table

leave the 2 to be carried on to the next number. The next number is found in the same manner, except using the second digit of 563: 6. We see that nine sixes (or nine times six), equals 54, the 4 is used as the second last digit, plus the 2, equalling 6, being carried forward from the previous number, in `9 * 563 = ___7'. The 5 is carried forward. Lastly, the 5 is multiplied by 9 in the table to get 45. The 5 being carried forward is added to 45, since there are no more numbers to be multiplied, resulting in the first digits of `9 * 563 = __67', being 50. So:

`9 * 563 = 5067'

To complete the question, the same procedure is carried out for the rest of the equations, looking under IV for 40, and II for 200. The only difference is that a zero must be added as the last digit for the second equation (40 * 563) as we are multiplying by 4, not 40, and two zeros for the third, and so on, for each number, as shown:

```
  9 * 563 =   5067
 40 * 563 =  22520
200 * 563 = 112600
249 * 563 = 140187
```

The results are added giving you the final answer: 249 * 563 = 140187.

Contrary to belief, this was a tremendous advance for the people of the era. Once caught on to the system, the time saved was tremendous. We must remember that the `bones were not pieces of paper, but rather, were made of wood or ivory and placed in a box to be more accessible and easy to use. The advantage to Napier's bones is that the carriers can actually be seen. To multiply 563 by 9, the carried 2 from the

Continued

Continued

multiplication of 3 * 9, can be mentally slid down the diagonal to line IX, to make 6 (4+2) for the second last digit, and slide the carried 5 down the diagonal to make 0 (5+5). We can then read off the result by adding the carried 1 to 4, equalling 5: 5067. For people constantly working with large numbers, Napier's bones were a life saver.

For the invention of logarithms, Napier had help from Henry Brigs (1561-1631). Together, on the British Isles, they worked on simplifying Napier's theories. Napier saw that every number can be expressed as a power of another number, called a base, thus being two ways of writing the same number. For example, 8 may be written as 2^3. According to Napier, this could be conveyed in three ways: two times two times two = 8; two to the power of three is 8; and logarithm 8 to the base 2 is 3. Napier and Brigs also discovered that to multiply 2 numbers, the logarithms are added, and to divide the logarithms are subtracted. For example:

$8 * 4 = 2^3 * 2^2 = 2^{(3+2)} = 2^5 = 32$
$8/4 = 2^3 / 2^2 = 2^1 = 2$

The above are simple calculations. With larger numbers, as in 23.9657 * 19.2739, the logarithms of the two numbers are looked up and added together. The result is looked up in a table of antilogarithms for the answer.

logarithm 23.9657 = 1.379590
logarithm 19.2739 = 1.234949
Sum of logarithms = 2.614539
Antilogarithm = 461.7299

The calculations of these logarithm and antilogarithm tables took Napier many pain-staking trials. In 1614, he published his table of logarithms in Mirific Logarithmorum Canonis Descriptio, explaining the steps he had climbed to this invention. To understand logarithms, we must recall two kinds of averages; arithmetic mean and geometric mean. Arithmetic mean is found by adding two numbers and dividing by 2, (6+4=10; 10/2=5). Geometric mean is found by multiplying two numbers and taking the square root, (6*4=24; $\sqrt{64}$=8). For any two known logarithm numbers, the logarithm is the arithmetic mean of the original two logarithms; the number is the geometric mean.[2] For example: first, using a base of 10, find the powers 10 must be raise to, to get the numbers 100 and 1000. The numbers are 2 and 3. Second, as the rule says, the unknown is the geometric mean of 100 and 1000, (100*1000=100,000; $\sqrt{100,000}$=316.226677). Third, the logarithm is the arithmetic mean of 2 and 3. (2+3=5; 5/2=2.5). The answer is thus; $10^{2.5}$=316.226677 or, the logarithm to base 10 of 316.226677 is 2.5.

The use of Napier's work was amazing. Scientists, engineers, and astronomers alike, thanked him for the new time-saving technique. By word of mouth, Johanne Kepler learned of Napier's discovery and used his logarithms to establish his three laws of planetary motion. Consequently, Isaac Newton also benefitted from Kepler's laws in revolutionizing the exact sciences. Napier's tables were used in schools around the world in math until the invention of calculators. The invention of logarithms greatly simplified calculations and the time spent

Continued

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doing them in the years to come.

In conclusion, the almost-forgotten John Napier, has made significant contributions to the mathematical pool of knowledge in 1996. His time-saving discoveries of Napier's bones, and logarithms significantly simplified the work of many professionals and students. The teaching of math without references to the contributing persons is a shame; the past is a vital key to the present.

II. SUMMARY

In summary; logarithms are numbers, usually grouped in a table, used to reduce complicated multiplications and divisions as well as addition, subtraction and problem solving. Similarily, Napier's Bones also reduce the time and complication in multiplication and division. Both the use of logarithms were used up to the invention of calculators. Although John Napier was very mathematically oriented he spent most of his time conveying the Anti-Christ campaign.

The limitations of the project included lack of resources. There are very few references on the history of mathematicians, thus verifying the theme of this essay, that math history truly is the neglected domain. Further investigations better libraries with more material is a possibility.

The mathematical knowledge encountered doing this essay included logarithm, exponents, and antilogarithms, all of which have been studied at sometime in my high school career.

III. REFERENCES

1. Dunham, William, Journey Through Genius, John Wiley and Sons Inc., U.S.A., 1990.
2. McLeish, John, Number, Ballantine Books, U.S.A., 1991.
3. Moffatt, Michael, The Ages Of Mathematics, Doubleday and Company Inc., U.S.A., 1977.
4. Solomon, Charles, Mathematics, Grosset and Dunlap, U.S.A., 1971.

Continued

Continued

Commentary — *This response would receive a score of* **25**

Defining the Project (3)
The student
- explains the purpose well and defines the topic
- sets the background and discusses the limitations of information
- provides a good introduction stating that "math taught today loses a great deal in not learning who contributed the time and effort for the major theories"

Conducting the Project (5)
The student
- could have given more extensive coverage to collecting information
- the information presented is well-organized
- acknowledges Napier's contributions to mathematics by recognizing the benefits of Napier's Bones to the works of Kepler and Newton

Analysis (4)
The student
- interprets the use of logarithms only vaguely
- makes an addition error in the third example

Mathematical Content (7)
The student
- lacks accuracy in some statements
- makes some incomplete interpretations
- makes appropriate use of mathematical symbols such as $\sqrt{\ }$, *, etc.

Communication (6)
The student
- has provided a well-organized and sequenced report
- states references used

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